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and fiscal competition

Jean J. Gabszewicz and Ornella Tarola



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Voie du Roman Pays, 34
B-1348 Louvain-la-Neuve
Belgium

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**Migration, wage differentials
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Jean J. GABSZEWICZ¹ and Ornella TAROLA²

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Abstract

We analyze the effects, on nonredistributive taxation and on migrations, of wage differentials existing between two countries (regions) differing by the size of their population. Residents, otherwise identical, are heterogeneous because they incur different migration costs. Each resident compares the post-tax amount of money at home with the one obtained abroad, including the cost of migration. The government in each country maximizes the tax product in order to provide the largest possible amount of public good. We first assume that the income of citizens are identical across countries. Then, we assume that wages differ from one country to the other. We prove the existence of an equilibrium for any configuration of wage and any different relative size of the countries (regions). Then, we compute and characterize the equilibrium for any set of parameters, size and wage differential.

Keywords: migration, taxation, wage differentials

¹ Université catholique de Louvain, CORE, B-1348 Louvain-la-Neuve, Belgium.
E-mail: jean.gabszewicz@uclouvain.be. This author is also member of ECORE, the association between CORE and ECARES.

² University of Rome "La Sapienza", DTE, Italy. E-mail: ornella.tarola@fastwebnet.it.

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1 Introduction

In this note, we analyze the effects on taxes and migrations of wage differentials existing between two countries (regions) differing by the size of their population. Migrations are populations' movements from some countries (jurisdictions) to others generated by migrants' feelings that life conditions would be better abroad than at home.

The lion's share of the literature on labor mobility and fiscal competition deals with the effects of redistributive taxation on migrations. It puts forward the fact that political institutions cannot intervene in their own jurisdiction as if they were living in isolation from the rest of the world, as individuals vote "with their feet" for their most preferred jurisdiction (Tiebout, 1956). In a way, redistributive policies give rise to a phenomenon of attraction/repulsion dichotomy: those individuals who benefit from redistribution are attracted to jurisdictions involved in redistribution, while those who are damaged are repelled. So, when putting in place redistributive policies in their own jurisdiction, political institutions are engaged in a sort of fiscal competition whose main consequences are not yet completely elucidated. This basic idea was introduced in the literature by Stigler (1957), Oates (1968), and Musgrave (1969). Then, mainly due to the European economic integration, in the past decades it got a renewed attention. Indeed, removal political and economic barriers between countries has determined an increasing mobility of factors, highly sensitive to different redistributive policies of governments members of the Union. See, for instance, Wildasin (1988, 1991, 2000, 2006), Myers (1990), Epple and Romer (1991), Wellisch (2000), Hansen and Kessler (2001), Piaser (2003), Puy (2003), Bierbrauer, Brett and Weymark (2011) among others.

While sharing with the mainstream literature some characteristics - in particular, the attraction/repulsion phenomenon -, the present paper is not directly concerned with the problem of redistribution. Starting from the fact that nations use taxation not only with a redistributive purpose, but also in order to finance other interventions of the Government, like financing the production of physical public goods, the activity of civil servants, the education of children, the defense of the territory, our analysis centers around the following question: *what is the impact of non-redistributive taxation policies on migration flows between countries and jurisdictions, differing in size and productive efficiency?*

Among the major elements influencing migration decisions of individuals, are the public facilities at home compared with those offered abroad, as well as the differentials in wages existing among countries. Regions with a rich panoply of public goods exert a strong attraction on the natives of countries (or regions) not so well endowed with such

public facilities. For instance, a large fraction of migrations from the developing countries to those belonging to the European Community could probably be explained by the presence in the latter of social protection and other social benefits, not available to such an extent in the former. Similarly, educational advantages of a country over another drain a significant fraction of the young citizens' population from the latter to the former. Of course, individuals are also attracted by private richness. The abundance and variety of private goods in a country is certainly a powerful attractor for citizens belonging to other, less endowed, regions. In particular, a large wage differential between two regions or two countries should clearly serve to set in motion migration movements from the lower to the higher wage region, simply because in market economies larger wages give access to more and better consumption.

Of course, migration decisions must also take into account the costs resulting from moving abroad, as well as the differential in taxes to be paid in each of them. The cost of moving abroad is heterogeneous across the population of residents. Some of them are strongly linked to their relatives living in their home country, while others are considerably more mobile, simply because they are less attached to the people living around them. National traditions, patriotism, historical origins and meteorological conditions constitute other values to be considered, with a varying influence across citizens of a given country. Accordingly, individuals placed otherwise in similar situations appear as heterogeneous in their willingness to move abroad to find better conditions in their economic environment. The differential in taxes is another crucial element influencing the decision to migrate. While a higher wage abroad plays as a powerful attractor for migrants, a larger tax pressure on the contrary operates as a strong repellent.

Also, while a lot of contributions is concerned with the problem of existence of an equilibrium under tax competition and capital mobility (see, for instance, Laussel and Le Breton 1998, Wildasin 1988, Fuest, Herbert and Mintz 2005, among others), nothing similar exists, to the best of our knowledge, for models with non redistributive tax competition and labor mobility. In the model considered below, we explicitly identify the unique Nash equilibrium in taxes corresponding to any set of exogenous parameters, namely, the size of each region and the wage differential (generated by different productive efficiencies) existing between them.

The above comments evoke the cornerstones of the simple model of labor mobility and fiscal competition studied in this note. There are two countries, home H and foreign F , with country F being assumed smaller in terms of its population. Citizens can freely move from one country

to the other. Residents otherwise identical are heterogeneous because they incur different migration costs. Each resident compares the post-tax amount of money obtained at home with the one obtained abroad, including the cost to be incurred due to migration. The government in each country maximizes the tax product in order to provide the largest possible amount of public good. Countries are assumed to play a two stage game. In the first stage, each government is assumed to set its tax, taking into consideration the possible migration flow initiated as a consequence of its fiscal pressure. In the second stage, residents in each country decide whether to stay in their own country or to migrate.

We start by considering the simplest situation where the *per capita* income of citizens in both jurisdictions is initially the same for each citizen and across regions. The main outcome of this section is that, at the equilibrium of the above-two stage game, only the size difference explains migration: the migration flow, when it exists, always goes from the larger country, H , to the smaller one, F . Furthermore the small country sets a smaller (and nonnull) tax than the larger whenever it is "very small". On the contrary, when its size gets closer to the larger one, the small country sets its tax equal to zero and keep it at that level, while the larger one plays its best reply against the zero tax strategy of the smaller country, and no further migration from H to F takes place anymore: this "corner" equilibrium with no migration occurs as long as the small country remains smaller than country H .

Then, we extend the analysis by considering that the firms inside each region own a same linear technology, and hire its citizens as workers, who get a wage equal to their marginal product. We assume that the linear technology differs across regions, entailing wages also different from one country to the other. We start by assuming that the larger region is more efficient than the small one. In this situation, the citizens living in the larger region, endowed with the better technology, always obtain a larger wage than in the small one. For this case, our main results are as follows. For small differentials in wages, migration takes place from the larger country H to the smaller country F . This migration is essentially motivated by the tax differential, the tax in country F being substantially weaker than in country H . However, in proportion as the wage differential increases, country F is forced to decrease its tax in order to keep its attractiveness on the citizens of its rival country. At some point of this process, its equilibrium tax becomes equal to zero. Then migration from H to F stops while the differential in wages goes on to increase. Suddenly, this differential becomes so important that it starts to become attractive for citizens of the smaller country to flee to country H . Of course, this reversal in the direction of migration keeps going on

as long as the differential of wages still increases. Thus, in a nutshell, migrations can be described as follows. For small differences in wages, the attractive force exerted by a smaller tax generates a migration flow from the larger to the smaller country. For a higher difference in wages, the flow is reverted because the difference in wages exerts an attractive force operating in the opposite direction.

Then, we consider the reverse case when the smaller country is more efficient than the larger one. In this situation, the large country benefits from being large while the smaller one enjoys a higher productive efficiency. Under this alternative assumption, there exists no interior equilibrium with migration from the smaller country to the larger one, while the reverse is true in a large domain of wage differentials. Nevertheless, migration from the smaller country can be observed whenever the larger country sets a zero tax at equilibrium, compensating thereby the smaller wage existing in it.

In the next section, we present the formal model. In section 3, we analyze tax competition assuming equal income for citizens in both regions. Section 4 and 5 extend the analysis to regions endowed with different linear technologies. Finally, a short conclusion summarizes our findings and opens avenues for further research.

2 The basic model

Consider two countries, H and F , which impose taxes on their residents. The population in each country is uniformly distributed over types and the set of types is represented in each country by the $[0,1]$ interval. Each type of resident is endowed with one unit of income. Assume without loss of generality that the population is larger in country H than in country F and denote by l_0 , $l_0 < \frac{1}{2}$, the population density in country F , and $1 - l_0$ the population density in H . Accordingly, country F is smaller than country H . We also assume that the population is ranked in each unit interval according to the *migration cost* to be paid when moving from one's own country to the other, which we assume to be equal to x for type x , $x \in [0, 1]$. Thus, migration cost is the only source of heterogeneity among the agents. In particular, we assume in the basic model that each citizen in each country is endowed initially with one unit of income. We denote by t_h (resp. t_f) the tax rate in country H (resp. F), $t_h, t_f \in [0, 1]$. The income tax revenue of the government writes as

$$(1 - l_0)t_h$$

in country H , and

$$l_0 t_f$$

in country F .

3 A tax game with the same income in each country

We assume that each Government maximizes its income tax revenue, given the tax chosen by the other Government, taking into account the future redistribution in the population resulting from the use of these taxes. This gives rise to a tax game, with countries H and F as *players*, taxes t_h and t_f as *strategies* and

$$\begin{aligned}\Pi_h(t_h, t_f) &= (1 - l_1(l_0))t_h \\ \Pi_f(t_h, t_f) &= l_1(l_0)t_f\end{aligned}\tag{1}$$

as *payoffs*, where $l_1(l_0)$ denotes the population in country F resulting from the redistribution between the populations after the use of the tax strategies t_h and t_f .

It is easy to identify the value of $l_1(l_0)$ according as migration takes place from H to F , or the reverse. First, notice that a necessary and sufficient condition for a migration to take place from H to F is that $t_h > t_f$. Indeed, all citizens y in country H belonging to the interval $[0, x]$ with $x = t_h - t_f$ prefer to move to country F due to the differential in taxes: staying at home these citizens y , with $y \leq x$, pay t_h while, moving abroad, they pay $t_f + y \leq t_h$, taking into account the migration cost y . Accordingly, the population of H in the interval $[0, t_h - t_f]$ migrates to country F , so that $x = t_h - t_f > 0$ is a necessary and sufficient condition to guarantee that migration takes place from H to F . Thus, the population $l_1(l_0)$ in country F after taxes is given by

$$l_1(l_0) = l_0 + (t_h - t_f)(1 - l_0).\tag{2}$$

A similar reasoning shows that, if migration takes place from F to H , which necessarily implies $t_f > t_h$, the population in country F after taxes is given by

$$l_1(l_0) = l_0(1 - (t_f - t_h)) = l_0(1 + (t_h - t_f)).\tag{3}$$

Thus, it is exclusively the differential in taxes which determines the direction and size, of migrations. Substituting (2) in system (1), we obtain

$$\begin{aligned}\Pi_h(t_h, t_f) &= (1 - (l_0 + (t_h - t_f)(1 - l_0)))t_h \\ \Pi_f(t_h, t_f) &= (l_0 + (t_h - t_f)(1 - l_0))t_f.\end{aligned}$$

Both these payoffs functions are concave in the domains $\{t_h : t_h > t_f\}$ and $\{t_f : t_f > t_h\}$, respectively.

Proposition 1 *The unique equilibrium of the tax game provides migration from country H to country F , and is given by*

$$t_f^* = \frac{l_0 + 1}{3(1 - l_0)}, t_h^* = \frac{2 - l_0}{3(1 - l_0)}; \quad (4)$$

Furthermore, $x(t_h^*, t_f^*) = \frac{(2l_0 - 1)}{3(l_0 - 1)}$ and $t_f^* \leq t_h^* \iff l_0 \leq \frac{1}{2}$.

Proof. Consider the tax game between countries H and F . Let (t_h^*, t_f^*) denote a Nash equilibrium of this game (if it exists) and assume first that a migration from H to F takes place at equilibrium. Then $t_h^* > t_f^*$, and all citizens of country H in the interval $[0, x]$ with $x = t_h^* - t_f^* > 0$ prefer to move to country F due to the differential in taxes. Accordingly, the population $l_1(l_0)$ in country F after taxes is given by

$$l_1(l_0) = l_0 + (t_h^* - t_f^*)(1 - l_0).$$

Since payoffs functions are concave in the domains $\{t_h : t_h > t_f^*\}$ and $\{t_f : t_f > t_h^*\}$, respectively, first order conditions must be satisfied at equilibrium and are given by

$$\frac{\partial \Pi_h(t_h, t_f)}{\partial t_h} = t_f - l_0 - 2t_h - l_0 t_f + 2l_0 t_h + 1 = 0$$

and

$$\frac{\partial \Pi_f(t_h, t_f)}{\partial t_f} = l_0 - 2t_f + t_h + 2l_0 t_f - l_0 t_h = 0.$$

Consequently, the candidate equilibrium values are necessarily given by the solution of the above system, namely,

$$t_f^* = \frac{l_0 + 1}{3(1 - l_0)}, t_h^* = \frac{2 - l_0}{3(1 - l_0)},$$

which satisfies the required constraints of positivity, and $x(t_h^*, t_f^*) = t_h^* - t_f^* > 0$ (we denote by $x(t_h^*, t_f^*)$ the value of x at equilibrium). It is easy to check that no value $t_h < t_f^*$, or $t_f < t_h^*$ could lead to a higher tax revenue, either for country H or for country F . Consequently (t_f^*, t_h^*) is a Nash equilibrium.

Now we show uniqueness. Assume that an equilibrium (t_h^*, t_f^*) would exist with migration from country F to country H . Then the inequality $t_f^* - t_h^* > 0$ must necessarily hold. Furthermore, all citizens of country F in the interval $[0, x(t_h^*, t_f^*)]$ with $x(t_h^*, t_f^*) = t_f^* - t_h^*$ prefer to move to country H due to the differential in taxes. Accordingly, the population $l_1(l_0)$ in country F after taxes is given by

$$l_1(l_0) = l_0(1 + t_h - t_f)$$

and payoffs are given by

$$\Pi_h(t_h, t_f) = (1 - l_0(1 + t_h - t_f))t_h$$

and

$$\Pi_f(t_h, t_f) = (l_0(1 + t_h - t_f))t_f.$$

A similar reasoning as above shows that the first order conditions

$$\begin{aligned} l_0 t_f - l_0 - 2l_0 t_h + 1 &= 0 \\ l_0 - 2l_0 t_f + l_0 t_h &= 0 \end{aligned}$$

must hold in the subdomain $\{(t_h, t_f) : t_f - t_h \geq 0\}$ so that the candi-

date equilibrium values must in particular solve the above linear system, namely, $t_f^* = \frac{1+l_0}{3l_0}$ and $t_h^* = \frac{2-l_0}{3l_0}$. But these values are not admissible since $x(t_h^*, t_f^*) = t_f^* - t_h^* < 0$, which contradicts our starting assumption according to which migration takes place from F to H at equilibrium. Accordingly, there exists no equilibrium to the tax game with migration from country F to country H . It is easy to check that neither $(t_h, t_f) = (0, 0)$, nor $(t_h, t_f) = (0, t_f)$ or $(t_h, 0)$ for some strictly positive values of t_h or t_f , can be an equilibrium. Thus the equilibrium identified above is the unique equilibrium of the tax game, which completes the proof of the proposition. Q.E.D

Thus, at the equilibrium of the above two stage game, migration is fully explained by the difference in size between the two regions: the migration flow always goes from the large country, H , to the smaller one, F . Notice that the equilibrium tax in the small country is smaller than in the larger one in the whole admissible range for l_0 ($[0, \frac{1}{2}]$). Also, both countries increase their tax in proportion as their sizes get closer and closer to each other, but the small country at a rate which is the double of the larger one. Paradoxically, it seems that this increase in taxes should be detrimental to both countries since it reinforces the incentives of natives for each country to flee to the other one. However, at the same time, this increase in taxes overcompensates the loss incurred in the national income due to migrants fleeing away outside the country. Now we examine the problem of migration when countries differ not only by size but also when wages are different in country H and country F . ■

4 Higher wages in country H

In order to make these wages endogenous, we suppose that each individual in each country is endowed with one unit of labour sold on a labour

competitive market. In country i , labour demand comes from a continuum of firms with a constant returns to scale production function $\alpha_i z$, $i = H, F$. Then, competitive wages w_h and w_f are given by $w_h = \alpha_H$ and $w_f = \alpha_F$. In the present section, we shall assume that country H has a higher productivity than country F , i.e, $w_h > w_f$. We examine the reverse case in section 5.

4.1 Migration from H to F : interior equilibria

Let us first consider migration from H to F . In order to determine the last citizen willing to leave from H to F , define x by the equality

$$w_h - t_h = w_f - t_f - x,$$

or

$$x = (t_h - t_f) - (w_h - w_f)$$

with $x > 0$, which is a necessary and sufficient condition to get a migration from H to F . Now it appears that the size and direction of migration not only depends on the difference between taxes, but also on the difference between the productivities, or equivalently, between the wages in the two countries. The resulting payoffs are given by

$$\begin{aligned}\Pi_f(t_h, t_f) &= t_f (l_0 + (1 - l_0) ((t_h - t_f) - (w_h - w_f))) \\ \Pi_h(t_h, t_f) &= t_h (1 - (1 - l_0) ((t_h - t_f) - (w_h - w_f)) - l_0).\end{aligned}$$

First order conditions obtain as: $\frac{\partial \Pi_h(t_h, t_f)}{\partial t_h} = t_f - l_0 - 2t_h - w_f + w_h - l_0 t_f + 2l_0 t_h + l_0 w_f - l_0 w_h + 1 = 0$ and $\frac{\partial \Pi_f(t_h, t_f)}{\partial t_f} = l_0 - 2t_f + t_h + w_f - w_h + 2l_0 t_f - l_0 t_h - l_0 w_f + l_0 w_h = 0$. The candidate equilibrium is given by the pair of strategies

$$\begin{aligned}t_f^* &= \frac{(w_h - w_f - l_0 + l_0 w_f - l_0 w_h - 1)}{3l_0 - 3} \\ t_h^* &= \frac{(l_0 + w_f - w_h - l_0 w_f + l_0 w_h - 2)}{3l_0 - 3}.\end{aligned}$$

In order of obtaining a migration from country H to country F , we need $1 > x(t_h^*, t_f^*) > 0$, which holds if, and only if

$$(w_h - w_f) < \frac{2l_0 - 1}{l_0 - 1}.$$

Furthermore it is necessary that $t_h^* > 0$, and $t_f^* > 0$. Easy computations show that both taxes are positive iff

$$t_f^* > 0 \iff (w_h - w_f) < \frac{(1 + l_0)}{1 - l_0} \quad (5)$$

$$t_h^* > 0 \iff (w_h - w_f) > \frac{l_0 - 2}{1 - l_0}, \quad (6)$$

where the last inequality is always satisfied since $l_0 < \frac{1}{2}$ and $w_h > w_f$. Notice also that, $\frac{(1+l_0)}{1-l_0} > \frac{2l_0-1}{l_0-1}$, so if $(w_h - w_f) < \frac{2l_0-1}{l_0-1}$ then *a fortiori* $(w_h - w_f) < \frac{(1+l_0)}{1-l_0}$. Thus we can conclude that, when the inequalities

$$0 < (w_h - w_f) < \frac{2l_0 - 1}{l_0 - 1} \quad (7)$$

hold, the pair of strategies (t_f^*, t_h^*) is an equilibrium. Thus we state

Proposition 2 *When the wage differential $w_h - w_f$ is positive and satisfies condition (7), the pair of strategies*

$$t_f^* = \frac{(w_h - w_f - l_0 + l_0 w_f - l_0 w_h - 1)}{3l_0 - 3}$$

$$t_h^* = \frac{(l_0 + w_f - w_h - l_0 w_f + l_0 w_h - 2)}{3l_0 - 3}$$

is a Nash equilibrium with migration from H to F . Furthermore, $t_f^ < t_h^*$.*

4.2 Migration from F to H : interior equilibria

Let us now consider migration from F to H . To this end, in accordance with the above section, we determine the marginal consumer x in F who is the last one to be willing to leave F for H , namely

$$w_h - t_h - x = w_f - t_f,$$

or

$$x = (w_h - w_f) + (t_f - t_h),$$

with $x > 0$, which is a necessary and sufficient condition to get a migration from F to H . From this it follows that population in F becomes equal to $l_0 (t_h - t_f + w_f - w_h + 1)$ while population in H obtains as $1 - l_0 (t_h - t_f + w_f - w_h + 1)$. Accordingly, payoffs now write as

$$\Pi_h(t_h, t_f) = (1 - l_0 (t_h - t_f + w_f - w_h + 1))t_h$$

and

$$\Pi_f(t_h, t_f) = l_0 (t_h - t_f + w_f - w_h + 1) t_f.$$

First order conditions are given by

$$\begin{aligned}\frac{\partial \Pi_h(t_h, t_f)}{\partial t_h} &= l_0 t_f - l_0 - 2l_0 t_h - l_0 w_f + l_0 w_h + 1 = 0 \\ \frac{\partial \Pi_f(t_h, t_f)}{\partial t_f} &= l_0 - 2l_0 t_f + l_0 t_h + l_0 w_f - l_0 w_h = 0.\end{aligned}$$

Solving the above equations in t_f and t_h respectively, we get as a candidate equilibrium the pair of strategies (t_f^{**}, t_h^{**}) defined by

$$\begin{aligned}t_f^{**} &= \frac{l_0 + l_0 w_f - l_0 w_h + 1}{3l_0} \\ t_h^{**} &= \frac{l_0 w_h - l_0 w_f - l_0 + 2}{3l_0}.\end{aligned}$$

First, let us check under which conditions $t_f^{**} > 0$, and $t_h^{**} > 0$, namely,

$$\begin{aligned}t_f^{**} > 0 &\Leftrightarrow (w_h - w_f) < \frac{(1 + l_0)}{l_0} \\ t_h^{**} > 0 &\Leftrightarrow (w_h - w_f) > \frac{(l_0 - 2)}{l_0}, \quad .\end{aligned}$$

where the last inequality is always satisfied as $(w_h - w_f) > 0 > \frac{(l_0 - 2)}{l_0}$. At this candidate equilibrium, migration would indeed take place from F to H iff $x(t_f^{**}, t_h^{**}) = \frac{2l_0 - l_0 w_f + l_0 w_h - 1}{3l_0} \in [0, 1]$ or $(w_h - w_f) > \frac{(1 - 2l_0)}{l_0}$. Accordingly, we conclude that (t_f^{**}, t_h^{**}) (with migration from F to H) is an equilibrium whenever

$$\frac{(1 - 2l_0)}{l_0} < (w_h - w_f) < \frac{(1 + l_0)}{l_0}. \quad (8)$$

Proposition 3 *When the wage differential $w_h - w_f$ is positive and satisfies condition (8), the pair of strategies*

$$\begin{aligned}t_f^{**} &= \frac{1}{3l_0} (l_0 + l_0 w_f - l_0 w_h + 1) \\ t_h^{**} &= \frac{1}{3l_0} (l_0 w_h - l_0 w_f - l_0 + 2)\end{aligned}$$

*is a Nash equilibrium with migration from F to H . Furthermore, $t_h^{**} > t_f^{**}$.*

Finally, we prove that, in the intervals $\left[0, \frac{2l_0-1}{l_0-1}\right]$ and $\left[\frac{1-2l_0}{l_0}, \frac{1+l_0}{l_0}\right]$, two equilibria cannot exist simultaneously. Another way to formulate the same question is whether there exist values of $l_0 (\leq \frac{1}{2})$, and wage differentials $w_h - w_f$ satisfying simultaneously the two conditions: (i) migration from H to F :

$$0 < (w_h - w_f) < \frac{2l_0 - 1}{l_0 - 1}$$

and (ii) migration from F to H :

$$\frac{(1 - 2l_0)}{l_0} < (w_h - w_f) < \frac{(1 + l_0)}{l_0}?$$

Denoting by $A = \frac{2l_0-1}{l_0-1}$, $B = \frac{(1-2l_0)}{l_0}$ and $C = \frac{(1+l_0)}{l_0}$, the two above conditions rewrite as

$$\begin{aligned} 0 < w_h - w_f &< A, \\ B < w_h - w_f &< C. \end{aligned}$$

The following inequalities are easily checked: $B > A, C > A, C > B$. So, we immediately derive from the above that the intervals $[0, A]$ and $[B, C]$ are disjoint. So, whenever $0 < (w_h - w_f) < A$, the equilibrium is (t_h^*, t_f^*) with migration taking place from country H to country F ; while for any $(w_h - w_f)$ satisfying $B < (w_h - w_f) < C$, the equilibrium is (t_h^{**}, t_f^{**}) with a migration flow from country F to country H .

4.3 Corner equilibria

Now we analyse whether in the "hole" between A and B , there exists an equilibrium (t_h^+, t_f^+) with migration taking place from H to F . To this end, we notice that the best reply, say $t_f(t_h)$, of country F when migration takes place from H to F is given by $t_f^+ = 0$ when $w_h - w_f = A$. Accordingly, using the first order condition $\frac{\partial \Pi_h(t_h, t_f)}{\partial t_h} = t_f - l_0 - 2t_h - w_f + w_h - l_0 t_f + 2l_0 t_h + l_0 w_f - l_0 w_h + 1 = 0$ and computing the value t_h^+ of the corresponding best reply function $t_h(t_f)$ of country H to country F against $t_f^+ = 0$, we obtain

$$t_h^+ = \frac{(1 + w_h - w_f)}{2},$$

with $t_h^+ > 0$. It immediately follows that the pair of strategies $(t_h^+, t_f^+) = \left(\frac{(1+w_h-w_f)}{2}, 0\right)$ is a candidate equilibrium in the range $[A, B]$. To complete the proof that it is indeed an equilibrium, we have still to show that there

exists no strictly positive value for t_f which could improve the payoff of country F , for any value of $w_h - w_f$ in the range $[A, B]$. To this end, first notice that $t_f(t_h)$, found in the scenario when migration takes place from H to F , is decreasing with the value of the wage differential $(w_h - w_f)$. Since, at $(w_h - w_f) = A$, $t_f(t_h) = t_f^+ = 0$, it follows that for any $(w_h - w_f)$ such that $A < (w_h - w_f) < B$, the best reply $t_f(t_h)$ would be strictly negative. Accordingly, there exists no strictly positive value for t_f which could improve the payoff of country F . This completes the proof that the pair $(t_h^+, t_f^+) = (\frac{(1+w_h-w_f)}{2}, 0)$ is, indeed, a Nash equilibrium. Finally, notice that, at this Nash equilibrium, $x(t_h^+, t_f^+)$ is strictly positive whenever $1 > w_h - w_f$ (res. negative whenever $1 < w_h - w_f$) so that migration (res. no migration) from H to F takes place.¹ Thus we can state:

Proposition 4 *When the wage differential $\frac{2l_0-1}{l_0-1} < w_h - w_f < \min(1, \frac{(1-2l_0)}{l_0})$, there exists a unique Nash equilibrium, with migration taking place from H to F , which is given by*

$$(t_h^+, t_f^+) = (\frac{(1 + w_h - w_f)}{2}, 0).$$

In summary, with higher wages in country H than country F , three scenarios can arise depending on the size of the differential in wages between the two countries. When the wage differential is small (smaller than A), migration takes place at equilibrium from country H to country F . This property seems counterintuitive at first sight, to the extent that the wage is higher in country H than in country F . However; the tax is larger in the latter than in the former, and the small wage differential in favour of country H is not attractive enough to compensate the repulsion effect generated by the higher tax in the larger country. The second scenario arises when the wage differential becomes more significant. Then the small country quotes a zero tax and, in spite of this, it can be still unable to attract more migrants from the larger country. Since the wage differential has become more favourable in country H , even a zero tax in country F cannot be attractive enough to release migration from the larger jurisdiction. Finally, when the wage differential is really important, the migration flow is reversed: citizens from the smaller jurisdiction are now attracted by the larger one due to a significant wage differential between the two countries.

¹It is easy to verify that $x(t_h^+, t_f^+) < 1$ always holds.

5 Higher wages in country F

5.1 Migration from H to F : interior equilibria

Let us first consider migration from H to F . In order to determine the marginal consumer in H who is the last willing to leave from H to F , define x by

$$w_h - t_h = w_f - t_f - x,$$

or

$$x = (w_f - w_h) - (t_f - t_h)$$

with $x > 0$, which is a necessary and sufficient condition to get a migration from H to F . Now the size and direction of migration not only depends on the difference between taxes, but also on the difference between the productivities, or equivalently, between the wages in the two countries. The resulting payoffs are given by

$$\begin{aligned}\Pi_f(t_h, t_f) &= t_f (l_0 + (1 - l_0) ((w_f - w_h) - (t_f - t_h))) \\ \Pi_h(t_h, t_f) &= t_h (1 - (1 - l_0) ((w_f - w_h) - (t_f - t_h)) - l_0).\end{aligned}$$

First order conditions are given by $\frac{\partial \Pi_h(t_h, t_f)}{\partial t_h} = t_f - l_0 - 2t_h - w_f + w_h - l_0 t_f + 2l_0 t_h + l_0 w_f - l_0 w_h + 1 = 0$ and $\frac{\partial \Pi_f(t_h, t_f)}{\partial t_f} = l_0 - 2t_f + t_h + w_f - w_h + 2l_0 t_f - l_0 t_h - l_0 w_f + l_0 w_h = 0$. The candidate equilibrium is given by the pair of strategies

$$\begin{aligned}t_f^* &= \frac{(w_h - w_f - l_0 + l_0 w_f - l_0 w_h - 1)}{3l_0 - 3} \\ t_h^* &= \frac{(l_0 + w_f - w_h - l_0 w_f + l_0 w_h - 2)}{3l_0 - 3}.\end{aligned}$$

In order of obtaining a migration from country H to country F , we need $1 > x(t_h^*, t_f^*) > 0$, which holds if, and only if $1 > (w_f - w_h) - (t_f^* - t_h^*) > 0$ or

$$\frac{2 - l_0}{1 - l_0} > w_f - w_h > \frac{1 - 2l_0}{l_0 - 1}$$

where $w_f - w_h > \frac{1 - 2l_0}{l_0 - 1}$ is always satisfied as $w_f - w_h > 0$ by assumption and $\frac{1 - 2l_0}{l_0 - 1} < 0$ as $l_0 < \frac{1}{2}$. Furthermore it is necessary that $t_h^* > 0$, and $t_f^* > 0$. Easy computations show that both taxes are positive iff

$$t_f^* > 0 \iff (w_f - w_h) > \frac{(1 + l_0)}{l_0 - 1}$$

$$t_h^* > 0 \iff (w_f - w_h) < \frac{(l_0 - 2)}{l_0 - 1},$$

where the first inequality is always satisfied since $l_0 < \frac{1}{2}$ and $w_f > w_h$. Thus we can conclude that, when the inequality

$$0 < (w_f - w_h) < \frac{2 - l_0}{1 - l_0} \quad (9)$$

holds, the pair of strategies (t_f^*, t_h^*) is an equilibrium with migration from H to F . Thus we state

Proposition 5 *When the wage differential $w_f - w_h$ satisfies condition (9), the pair of strategies*

$$t_f^* = \frac{(w_h - w_f - l_0 + l_0 w_f - l_0 w_h - 1)}{3l_0 - 3}$$

$$t_h^* = \frac{(l_0 + w_f - w_h - l_0 w_f + l_0 w_h - 2)}{3l_0 - 3}$$

is the unique Nash equilibrium with migration from H to F .

5.2 Migration from F to H

Let us now consider migration from F to H . To this end, we determine the marginal consumer x in F who is the last one to be willing to leave F for H , namely

$$w_h - t_h - x = w_f - t_f,$$

or

$$x = (w_h - w_f) + (t_f - t_h),$$

with $x > 0$, which is a necessary and sufficient condition to get a migration from F to H . From this it follows that population in F becomes equal to $l_0 (t_h - t_f + w_f - w_h + 1)$ while population in H obtains as $1 - l_0 (t_h - t_f + w_f - w_h + 1)$. Accordingly, payoffs now write as

$$\Pi_h(t_h, t_f) = (1 - l_0 (t_h - t_f + w_f - w_h + 1))t_h$$

and

$$\Pi_f(t_h, t_f) = l_0 (t_h - t_f + w_f - w_h + 1) t_f.$$

First order conditions are given by

$$\begin{aligned}\frac{\partial \Pi_h(t_h, t_f)}{\partial t_h} &= l_0 t_f - l_0 - 2l_0 t_h - l_0 w_f + l_0 w_h + 1 = 0 \\ \frac{\partial \Pi_f(t_h, t_f)}{\partial t_f} &= l_0 - 2l_0 t_f + l_0 t_h + l_0 w_f - l_0 w_h = 0.\end{aligned}$$

Solving the above equations in t_f and t_h respectively, we get as a candidate equilibrium the pair of strategies (t_f^{**}, t_h^{**}) defined by

$$\begin{aligned}t_f^{**} &= \frac{1}{3l_0} (l_0 + l_0 w_f - l_0 w_h + 1) \\ t_h^{**} &= \frac{1}{3l_0} (l_0 w_h - l_0 w_f - l_0 + 2)\end{aligned}$$

where $t_f^{**} > 0$ iff $w_f - w_h > \frac{-1-l_0}{l_0}$, and $t_h^{**} > 0$ iff $w_f - w_h < \frac{2-l_0}{l_0}$. Notice however that $x(t_f^{**}, t_h^{**}) < 0$. Consequently,

Proposition 6 *There exists no interior equilibrium with a positive flow of migrants from F to H when $w_f > w_h$.*

5.3 Corner equilibria

So, it remains to check whether there exists an equilibrium with a positive flow of migrants from F to H when in this latter country, the fiscal burden is equal to zero (corner equilibrium). To this end, we assume that $t_h^{++} = 0$ and examine the value of the best reply function of F against the strategy 0, namely $t_f(t_h^{++} = 0)$. Accordingly, using the first order condition $\frac{\partial \Pi_f(t_h, t_f)}{\partial t_f} = l_0 - 2l_0 t_f + l_0 t_h + l_0 w_f - l_0 w_h = 0$ and computing the value $t_f^{++}(t_h^{++} = 0)$, we get

$$t_f^{++} = \frac{(1 - w_h + w_f)}{2},$$

with $t_f^{++} > 0$. Notice also that $1 > x(t_f^{++}, t_h^{++}) > 0$ iff $1 > w_f - w_h$. It immediately follows that the pair of strategies

$$(t_h^{++}, t_f^{++}) = (0, \frac{(1 - w_h + w_f)}{2})$$

is a candidate equilibrium. To complete the proof that it is indeed an equilibrium, we have still to show that $t_h^{++} = 0$ is a best reply for country H against t_f^{++} . Evaluating $t_h^{++}(t_f^{++} = \frac{(1-w_h+w_f)}{2})$, we obtain that the strategy 0 is, indeed, the best reply against $t_f^{++} = \frac{(1-w_h+w_f)}{2}$. Accordingly, whenever $1 > w_f - w_h$ (res. $1 < w_f - w_h$) the pair of strategies (t_h^{++}, t_f^{++}) is a Nash equilibrium of the game with (res. without) a positive flow of migrants from F to H .

Proposition 7 *For any wage differential $1 > w_f - w_h > 0$, there exists a unique Nash equilibrium, with migration from country F to country H , given by the pair of strategies*

$$(t_h^+, t_f^+) = (0, \frac{(1 - w_h + w_f)}{2}).$$

When, on the contrary, $w_f - w_h \geq 1$, (t_h^+, t_f^+) is a Nash equilibrium without migration in neither direction.

The two above propositions are perfectly in line with intuition. No equilibrium from the small to the large country exists with a migration flow from the smaller country to the larger one: now the wage is higher in the smaller country. Of course, even the larger country can manipulate its tax in order to make it as attractive as possible, i.e. by setting its tax equal to zero, generating thereby a corner equilibrium possibly with a positive migration from the smaller country to the larger one, in spite of the smaller wage paid in the latter.

6 Conclusion

Given the relative sizes of the jurisdictions, the direction and the size of labor mobility in our model depends on two interrelated and opposite forces: the differential in wages and the differential in taxes. While the differential in wages is fully determined by the technological superiority of a country over the other, the difference between taxes is determined endogenously at equilibrium, and depends both on the differential in wages and on the different size of the two jurisdictions. As described in the introduction, and confirmed formally in section 4, when technological efficiency is larger in country H than in country F , migration takes place, for small differentials in wages, from the larger to the smaller country. This migration is essentially motivated by the tax differential, the tax pressure in country F being substantially weaker than in country H . However, when the wage differential increases due to increasing difference in productive efficiency, country F has to decrease its tax in order to remain attractive for the citizens of the rival country. At some point of this process, its equilibrium tax becomes equal to zero. Then migration from H to F stops while the differential in wages goes on to increase. Beyond some value of this differential, it starts to be so important that it becomes attractive for citizens of the smaller country to flee to country H . Of course, this reversal in the direction of migration keeps going on as long as the differential of wages still increases. Thus, equilibrium migrations evolve as follows. For small differences in wages, the attractive force exerted by a smaller tax generates a migration flow

from the larger to the smaller country. For a higher difference in wages the flow is reverted because the difference in wages exerts an attractive force operating in the opposite direction. In the middle, for intermediate values of the wage differential, the solution takes the form of a corner equilibrium, in which the small country sets a null tax while no migration takes place in either direction. As described in section 5, when technological efficiency is higher in the smaller country than in the other, no migration flow can be observed from the smaller country to the larger one, except when the latter sets a zero tax in order to compensate the lower wage at home.

Probably our model could be improved in order to analyze more traditional questions raised by the mobility of the labor force, resulting in particular from redistributive policies. This new formulation should consider the impact of taxes on a population whose members, contrary to the present approach, would differ by their initial income or by their productivity. Another natural extension of the present research consist in examining how migrations take place as a function of the relative size of the two countries, for a fixed wage differential of wages between the two countries. This would consist in replicating the analysis contained in section 3, where the differential of wages (income) is assumed to be equal to zero, but where the relative size of the small country is varying between 0 and 1. The extension would consider any value for the differential of wages. We did not do it ourselves because we found it would be a simple exercise for the interested reader.

Also a very restricted set of theoretical questions raised by migrations has been considered in this note. In particular, the European countries seem today by far more concerned with repelling migrants than attracting them. The main reason for this inversion of motivation seems to be related to the disastrous economic situation that we presently meet in Europe, generating in particular an important unemployment in western countries. In such periods of the business cycle, governments generally prefer to protect their labour markets from migrants' invasion. Of course, in more favourable periods, the reverse is true and migrants are much better welcome! It would be fascinating to analyze how rival countries solve the problem of mutual rejection of migrants unable to be accepted anywhere. But this is a completely different story than the one raised in this paper and which could be an interesting topic for further research. Nevertheless we feel that our endeavor is fully justified by the clear and simple results obtained using the present formulation.

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